وبسایت اینترنتی جزوه و تمرین

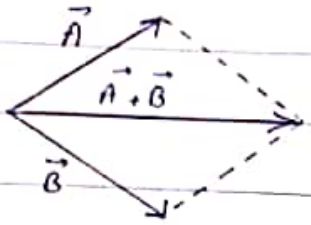
@JozveTamriN

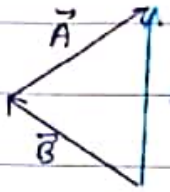
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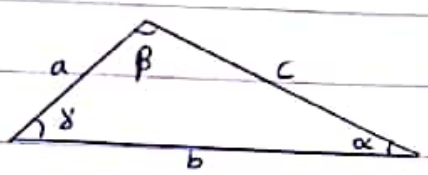
استاتیک و مقاومت مصالح

بردارها:





* $\vec{B}$ منفی است در تفاضل

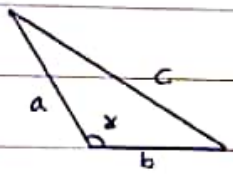


$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

$$c^2=a^2+b^2+2ab\cos\gamma$$

$$0<\gamma<180$$

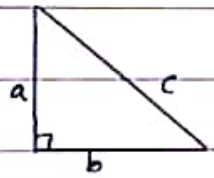
γ ← حاده



$$c^2=a^2+b^2-2ab\cos\gamma$$

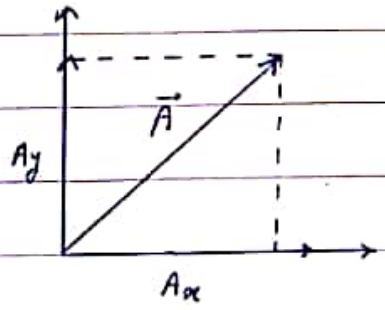
(منفی ← مثبت)

γ ← منفرجه



$$c^2=a^2+b^2-2ab\cos\gamma$$

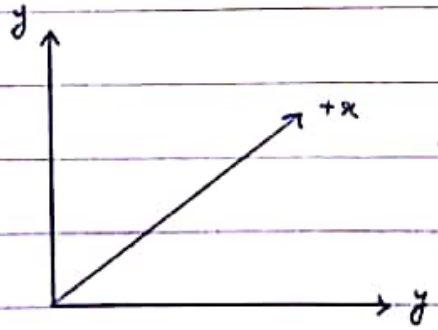
($\cos\gamma$: 0)



$$A=A_x i+A_y j$$

$$B=B_x i+B_y j$$

$$A+B=(A_x+B_x)\hat{i}+(A_y+B_y)\hat{j}$$



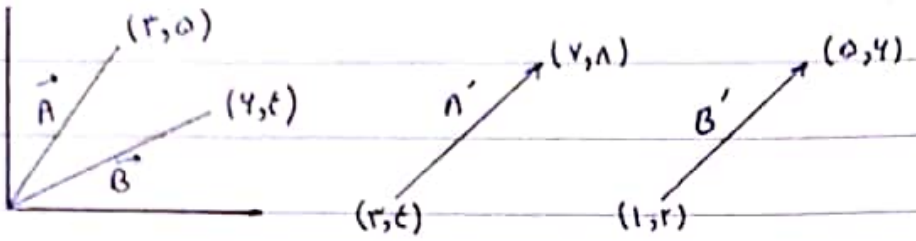
$$(A+B)=\sqrt{(A+B)^2+(A_y+B_y)^2}$$

$$\alpha=\gamma\left(\frac{A_y+B_y}{A_x+B_x}\right)$$

$$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$$

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مثال)



① : $A=(۳,۵) \rightarrow A_x=۳$ $A_y=۵$ $(A_x+B_x)\hat{i}$ $\vec{A}+\vec{B}=۹\hat{i}+۹\hat{j}$

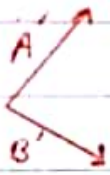
$B=(۴,۴) \rightarrow B_x=۴$ $B_y=۴$ $(A_y+B_y)\hat{j}$ $\tan(A+B)=\tan^{-1}\frac{۹}{۹}=۱ \Rightarrow ۴۵^\circ$

؟

② : $A'=(۷,۸)-(۳,۴)=(۴,۴)$ $A'+B'=(۴+۴)\hat{i}+(۴+۴)\hat{j}=(۸,۸)$

$B'=(۵,۷)-(۱,۳)=(۴,۴)$ $\tan(A'+B')=\frac{۸}{۸}=۱ \Rightarrow ۴۵^\circ$

ضرب :



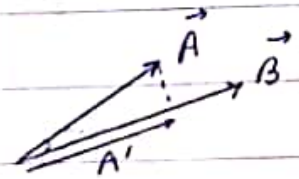
$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$

$\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$ ضرب نقطه‌ای (داخلی):

$A.B=A_xB_x+A_yB_y+A_zB_z$

ضرب خارجی : $A\times B=\begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}=(A_yB_z-A_zB_y)\hat{i}-(A_xB_z-A_zB_x)\hat{j}+$

$(A_xB_y-A_yB_x)\hat{k}$



مثال) تصویر $\vec{A}$ روی $\vec{B}$ ؟

$A'=\frac{\vec{A}.\vec{B}}{|\vec{B}|^2}.\vec{B}$

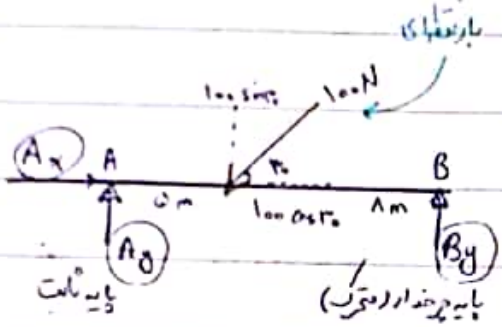
$|A'|=|A|\cos\alpha \xrightarrow{\times\frac{|B|}{|B|}} |A'|=\frac{|A||B|\cos\alpha}{|B|} \Rightarrow |A'|=\frac{A.B}{|B|}$

$A'(|A'|,\frac{B}{|B|})$ جهت دارد $\rightarrow \vec{A'}=\frac{A.B}{|B|^2}\vec{e}$

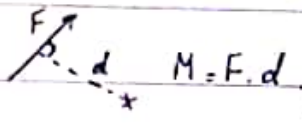
برای جهت دار کردن ضرب برداری

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تعیین عکس العمل پایه‌های تیرها؟



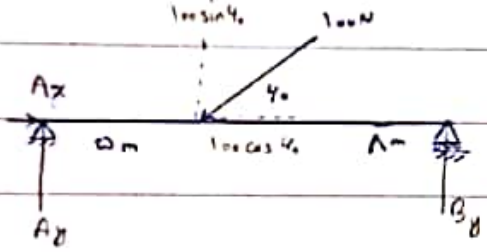
معادلات تعادل
$$\begin{cases} \sum F_x = 0 \rightarrow A_x - 100\cos 30 = 0 \\ \sum F_y = 0 \rightarrow A_y + B_y - 100\sin 30 = 0 \\ \sum M_A = 0 \rightarrow -B_y \times 13 + 100\sin 30 \times 5 = 0 \end{cases}$$



گشتاور = نیرو × (فاصله عمودی)

عمودی

مثال: تیر را حل کنید (عکس العمل پایه‌های آن را بدست آورید؟)



تعریف ممان: اگر یک نیرو یا استاد آن از یک نقطه عبور کند ممان آن صفر خواهد بود.

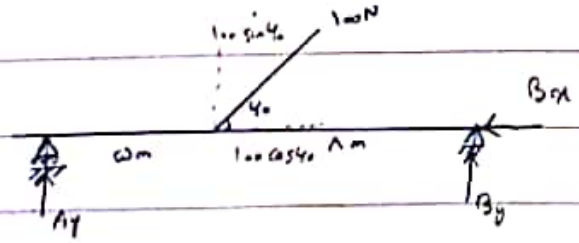
معادلات تعادل
$$\begin{cases} \sum F_x = 0 \rightarrow A_x - 100\cos 40 = 0 \rightarrow A_x = 80\,N \\ \sum F_y = 0 \rightarrow A_y + B_y - 100\sin 40 = 0 \rightarrow A_y + B_y = 84{,}4\,N \\ \sum M_A = 0 \rightarrow (-B_y \times 13) + (100\sin 40 \times 5) = 0 \rightarrow B_y = 24{,}3\,N \\ \sum M_B = 0 \rightarrow (A_y \times 13) - 100\sin 40 \times 8 = 0 \quad 3A_y = 100 \times \frac{\sqrt{3}}{2} \times 8 \rightarrow A_y = 60{,}3\,N \\ \sum M_C = 0 \rightarrow A_y \times 5 - B_y \times 8 = 0 \rightarrow 5A_y = 8B_y \rightarrow A_y = \frac{8}{5}B_y \end{cases}$$

$A_y + B_y = 84{,}4 \rightarrow \frac{8}{5}B_y + B_y = 84{,}4 \rightarrow B_y = 24{,}3$

$A_y = 60{,}3$

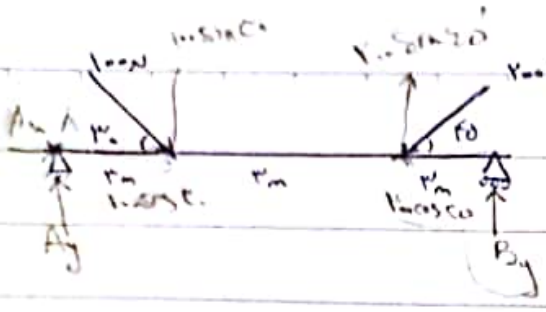
دقت شود همه معادلات درست است!

* سؤال مثل پایه‌های برعکس:



معادلات تعادل
$$\begin{cases} \sum F_x = 0 \rightarrow -B_x - 100\cos 40 = 0 \rightarrow B_x = -80 \\ \sum F_y = 0 \rightarrow B_y + A_y - 100\sin 40 = 0 \rightarrow B_y + A_y = 84{,}4 \\ \sum M_B = 0 \rightarrow -A_y \times 13 + 100\sin 40 \times 8 = 0 \rightarrow A_y = 60{,}3 \quad B_y = 24{,}3 \end{cases}$$

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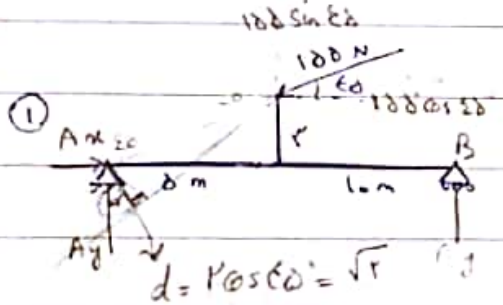


$$\begin{cases} \Sigma F_x = 0 \rightarrow \\ \Sigma F_y = 0 \rightarrow \\ \Sigma M \end{cases}$$

$$\Sigma F_x = 0 \rightarrow A_x + 100\cos 30 - 200\cos 50 = 0 \rightarrow A_x = -44.4 + 128.6 = 84.2$$

$$\Sigma F_y = 0 \rightarrow A_y + B_y - 100\sin 30 - 200\sin 50 = 0 \rightarrow A_y + B_y = 191.6$$

$$\circlearrowleft \Sigma M_A = 0 \rightarrow B_y \times 9 - 200\sin 50 \times 4 - 100\sin 30 \times 3 = 0 \rightarrow$$



حل:

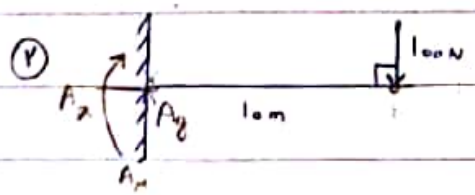
$$\Sigma F_x = 0 \rightarrow A_x - 100\cos 50 = 0 \rightarrow A_x = 109.4$$

$$\Sigma F_y = 0 \rightarrow A_y + B_y - 100\sin 50 = 0 \rightarrow A_y + B_y = 109.4$$

$$\circlearrowleft \Sigma M_A = 0 \rightarrow B_y \times 10 - 100\sin 50 \times 0 - 100\cos 50 \times 2$$

$$\rightarrow B_y = 0.128 \Rightarrow A_y = 109.6$$

$$\circlearrowleft \Sigma M_D = 0 \rightarrow A_y \times 0 - 100\cos 50 \times 3 - B_y \times 10 = 0 \rightarrow$$

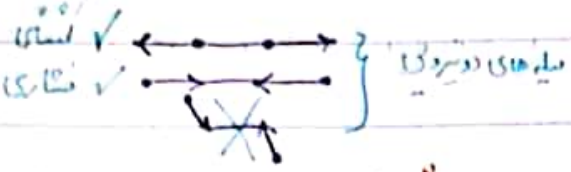


$$\Sigma F_x = 0 \rightarrow A_x - 0 = 0 \rightarrow A_x = 0$$

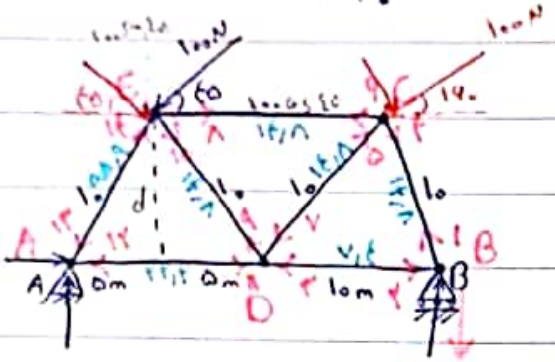
$$\Sigma F_y = 0 \rightarrow A_y - 100 = 0 \rightarrow A_y = 100$$

$$\Sigma M_A = 0 \rightarrow A_M + 10(100) = 0 \rightarrow A_M = -1000$$

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مثال:



$\Sigma F_x = 0 \rightarrow A_x - 100\cos 60 = 0 \rightarrow A_x = 100\cos 60 = 50\ N$

$\Sigma F_y = 0 \rightarrow A_y + B_y - 100\sin 60 = 0 \rightarrow A_y + B_y = 100\sin 60$

$\Sigma M_A = 0 \rightarrow B_y \times 20 - 100\sin 60 \times 5 + 100\cos 60 \times 8.4$?

$B_y = -12.9 \Rightarrow A_y = 87.4$

$d' = 100 - 20 - 50 \rightarrow d = 8.4$

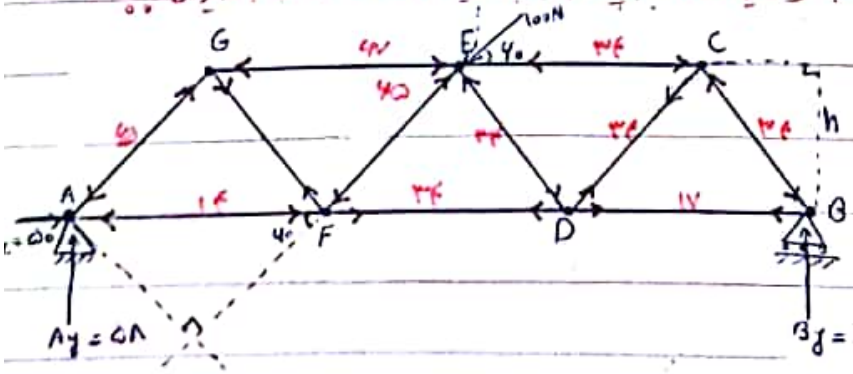
B:
$\Sigma F_x = 0 \rightarrow DB - BC\cos 40 = 0 \rightarrow DB = 7.4$
$\Sigma F_y = 0 \rightarrow BC\sin 40 = 12.9 \rightarrow BC = 16.8$

C:
$\Sigma F_x = 0 \rightarrow -EC + DC\cos 40 + 16.8\sin 40 = 0 \rightarrow DC = EC = 16.1$
$\Sigma F_y = 0 \rightarrow DC\sin 40 = 16.8\sin 40 \rightarrow DC = 16.8$

D:
$\Sigma F_x = 0 \rightarrow AD - ED\cos 40 - 7.4 - 16.8\cos 40 = 0 \rightarrow AD = 32.1$
$\Sigma F_y = 0 \rightarrow ED\sin 40 - 16.8\sin 40 = 0 \rightarrow ED = 16.8$

A:
$\Sigma F_x = 0 \rightarrow 70 - AE\cos 40 - 32.2 = 0 \rightarrow AE = 90.4$
$\Sigma F_y = 0 \rightarrow 87.4 - AE\sin 40 = 0 \rightarrow AE = 94.9$ ≃

* تمرین: خرپای بالا را با نیروی قرینگ حل کنید / عدد با هر دو نیرو حساب کنید / عدد با دو نیروی راست / عدد با دو نیروی چپ ؟



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•A
$\sum F_x = 0 \rightarrow A_x = 100\cos 40$
$\uparrow \sum F_y = 0 \rightarrow A_y + B_y = 100\sin 40$
$\circlearrowleft \sum M_A = 0 \rightarrow -B_y \times 10 + 100\sin 40 \times 7/0 - 100\cos 40 \times 5\sin 40 = 0$

$\circlearrowleft \sum M_B = 0 \rightarrow A_y \times 10 - 100\sin 40 \times 7/0 - 100\cos 40 \times 5\sin 40 = 0$

$-B_y \times 10 + 100 \times 5 \sin 40 = 0$

$A_y \times 5 = 0$

$B_y = \frac{64 \times 7/0 - 64 \times 2/0}{10} = 28/7$

$A_y = \frac{64 \times 10}{10} = 51/4$

•B
$\sum F_x = 0 \rightarrow -BD + BC\cos 40 = 0 \rightarrow BD = 17$
$\sum F_y = 0 \rightarrow 29 - BC\sin 40 = 0 \rightarrow BC = 45$

•C
$EC - 45(1/2) - 45(1/2) = 45$
$\sum F_x = 0 \rightarrow EC - BC\cos 40 - CD\cos 40 = 0$
$\sum F_y = 0 \rightarrow BC\sin 40 - CD\sin 40 = 0$
$\Rightarrow CD = 45$

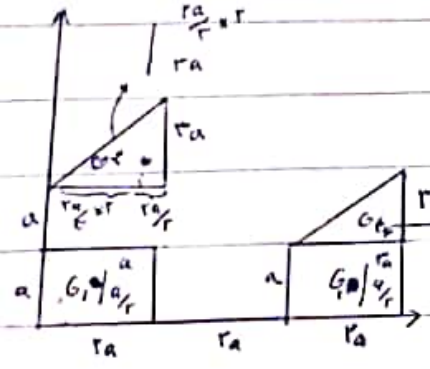
•D
$\sum F_x = 0 \rightarrow \overbrace{DB}^{17} + \overbrace{ED}^{17}\cos 40 - \overbrace{FD}^{17}\cos 40$
$\sum F_y = 0$
$-ED\sin 40 + CD\sin 40 = 0 \rightarrow ED = 45 \rightarrow FD = ED = 45$

•E
$\sum F_x = 0 \rightarrow GE - 25 - \overbrace{100}^{50}\cos 40 + \overbrace{FE}^{43}\cos 40 - 25\cos 40 = 0$
$\sum F_y = 0 \rightarrow \underbrace{-100\sin 40}_{-50\sqrt{3}} + FE\sin 40 + \underbrace{25\sin 40}_{17\sqrt{3}} = 0$
$\Rightarrow FE = 70$
$\Rightarrow GE = 27$

•F
$\sum F_x = 0 \rightarrow FD + FA - (FE\cos 40 - FG\cos 40)$
$\sum F_y = 0 \rightarrow FG\sin 40 - 70\sin 40 = 0 \Rightarrow FG = 70$

$70 \times 1/2 - FA \times 1/4 - FG \times 1/4 \rightarrow FA = \frac{70}{2} + 35/0 - 50 \Rightarrow FA = 10$

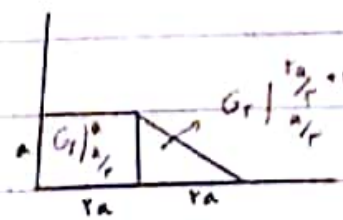
تعیین مرکز ثقل مرکب متشکل، سطح دایره:



$\bar{X} = \frac{\sum x_i A_i}{\sum A_i} = \frac{x_1 A_1 + x_2 A_2}{A_1 + A_2} = \frac{5a \times r a^2 + \frac{14a}{3} \times r a^2}{r a^2 + r a^2} = \frac{29a}{6}$

$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2} = \frac{3/2 \times r a^2 + r a \times r a^2}{5 a^2} = \frac{V a}{8}$

مثال:

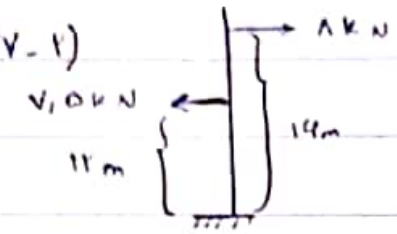


$\bar{x} = \frac{a \times r a^2 + \frac{8a}{3} \times a^2}{7 a^2} = \frac{14}{9} a$

$\bar{y} = \frac{\frac{a}{3/2} \times r a^2 + \frac{a}{3/2} \times a^2}{7 a^2} = \frac{4}{9} a$

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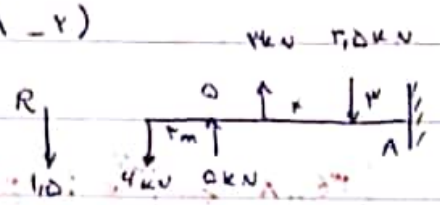
۴۷-۱)



$R = 0,5\ kN$

$\Sigma M_A = R \times d \rightarrow 8 \times 14 - 7,5 \times 12 = 24 \rightarrow d = 4$

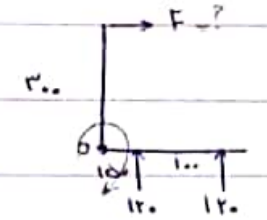
۴۸-۲)



$\Sigma M_A = R \times d \rightarrow -1,5 \times b = 21 + 60 - 10,5 - 72$

$\rightarrow -1,5b = -16,5 \rightarrow b = 11$

۷۱-۲)



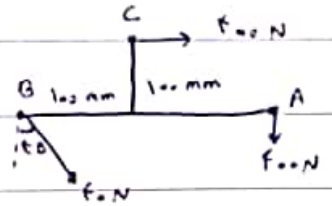
$\Sigma M_O = R \times d$

$\Sigma M_O = \ldots F \times 300 - 120 \times 150 - 120 \times 250$

$\rightarrow 18000 + 30000 = F \times 300 \rightarrow F = 160$

۷۱-۲) $\Sigma M_{\ldots} = 0 \rightarrow -80 \times 600 - 120 \times 200 + P \sin 70 \times 250 \ldots \rightarrow 224 = P \sin 70 \rightarrow P = 238,3$

۷۴-۲)

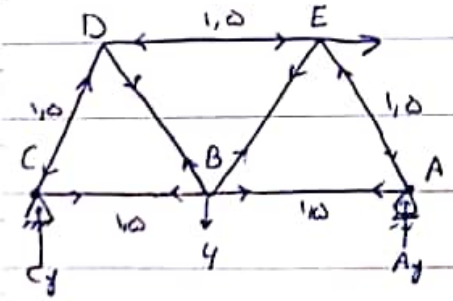


$\begin{cases} \Sigma F_x = R_x \rightarrow 600 + 600 \times \frac{\sqrt{2}}{2} \Rightarrow R_x = 949,8 \\ \Sigma F_y = R_y \rightarrow -600 - 600 \times \frac{\sqrt{2}}{2} \Rightarrow R_y = -849,8 \end{cases}$

$R_T = \sqrt{R_x^2 + R_y^2} \Rightarrow 940,7$

$\tan\theta = \frac{R_y}{R_x} = -1 \rightarrow \theta = -45^\circ$

۳-۴)



$\begin{cases} \Sigma F_x = 0 \rightarrow F - C_x = 0 \rightarrow C_x = 4kN \\ \Sigma F_y = 0 \rightarrow -4 + A_y + C_y = 0 \rightarrow C_y = 1,3 \\ \Sigma M_C = \ldots \rightarrow A_y \times 2 + 4 \times 1,5 + F \times 1,2 = 4,7 \rightarrow A_y \end{cases}$

$A \Rightarrow \Sigma F_x = 0 \rightarrow AE \times \cos 40 - AB = 0 \rightarrow \Sigma F_y = 0$

$\rightarrow 1,72 - AE \times \sin 40 \rightarrow \begin{cases} AE = 0,5 \\ AB = 1,75 \end{cases}$

$E \Rightarrow \Sigma F_x = \ldots \rightarrow 1 + DE - 0,5 \times \cos 40 - BE \times \cos 70 = 0 \rightarrow DE = 1,5$

$\Sigma F_y = \ldots \rightarrow BE = AE = 0,5$

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$B \Rightarrow \sum F_x = 0 \rightarrow 2,8 - BC + 5,0 \cos 40 - \overset{4,0}{DB} \cos 40 = 0 \rightarrow -BC = 4,0 \times \frac{1}{2} - 5,0 \times \frac{1}{2} - 2,8 \Rightarrow BC = 4,7$

$\sum F_y = 0 \rightarrow DB + BE = \frac{4}{\sin 40} \rightarrow DB = 1,0$

$C \Rightarrow \sum F_x = 0 \rightarrow CB - Cx - CD \cos 40 = 0 \rightarrow CB = 4,7$ ✓

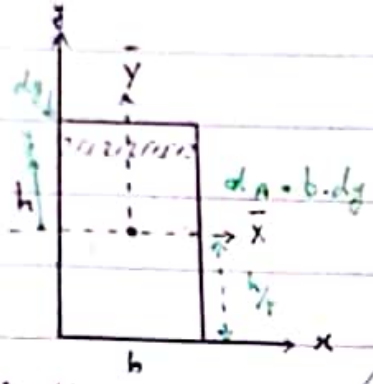
$\sum F_y = 0 \rightarrow CD \sin 40 = 1,3 \rightarrow CD = 1,0$

تمرین: ص $\overset{\text{فصل}}{\text{}}$ ۳۴ ← ۸



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لنگر دوم سطح - ممان دوم سطح:



$$I_x \cdot I_{xx} = \int y^2 dA = \int_{-h/2}^{h/2} y^2 \cdot b\, dy = b \frac{y^3}{3}\Big|_{-h/2}^{h/2} = \frac{2b}{3}\left(\frac{h}{2}\right)^3 =$$

$$I_y = I_{yy} = \int x^2 dA$$

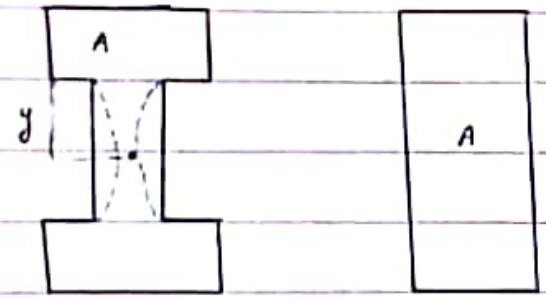
$$\frac{bh^3}{12} + \left(\frac{h}{2}\right)^2 \cdot bh = \frac{bh^3}{12} + \frac{bh^3}{4} = \boxed{\frac{bh^3}{3}}$$

$$I_{xx} = I_{\bar{x}\bar{x}} + \Delta x^2 \cdot A$$

$$A = bh$$

$$\Delta x = \frac{h}{2}$$

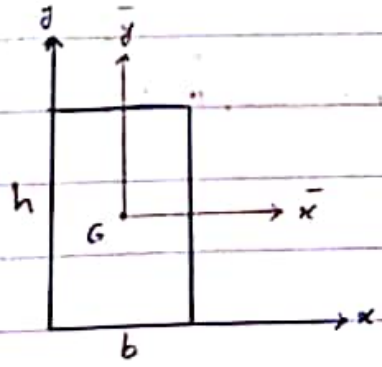
$$J = I_{xx} + I_{yy}$$



* در مقطع I به دلیل این که مقدار کمتری از اطراف مرکز ثقل وجود دارد، I کمتر

بیشترین میزان

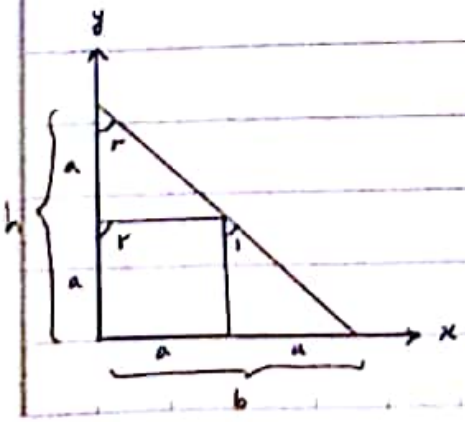
مثال ۱) ممان دوم را بدست آورید؟



$$I_{\bar{x}\bar{x}} = \frac{bh^3}{12}$$

$$I_{\bar{y}\bar{y}} = \frac{hb^3}{12}$$

مثال ۲) ممان دوم را بدست آورید؟

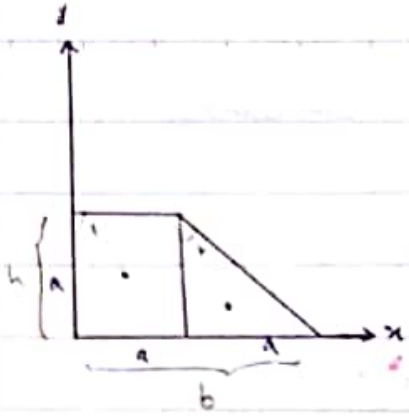


$$I_{xx} = \frac{bh^3}{36} \longrightarrow \frac{(2a)(2a)^3}{36} = \frac{16a^4}{36}$$

$$I_{yy} = \frac{hb^3}{36} \longrightarrow \frac{(2a)(2a)^3}{36} = \frac{16a^4}{36}$$

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مثال ۳) مان دوم را بدست آورید!



$$\bar{x} = \frac{\frac{1}{3}a^3 + \frac{1}{2}a \cdot \frac{a^2}{2}}{a^2 + \frac{a^2}{2}} = \frac{7}{9}a$$

$$\bar{y} = \frac{\frac{a}{3} \times a^2 + \frac{a}{2} \cdot \frac{a^2}{2}}{a^2 + \frac{a^2}{2}} = \frac{5}{9}a$$

$$\Delta x_1 = \frac{a}{2} - \frac{5}{9}a = \frac{1}{18}a \;\; \Delta y_1 = \frac{7}{9}a - \frac{a}{2} = \frac{5}{18}a$$

$$\Delta x_2 = \frac{2}{3}a - \frac{a}{2} = \frac{3a}{12} = \frac{a}{4} \;\; \Delta y_2 = \frac{5}{6}a - \frac{7}{9}a = \frac{15}{27}a = \frac{5a}{9}$$

$$I_{\bar{x}\bar{x}} = I_{\overline{x_1x_1}} + I_{\overline{x_2x_2}} = I_{x_1x_1} + A_1\Delta x_1^2 + I_{x_2x_2} + A_2\Delta x_2^2 = \frac{a \times a^3}{12} + a^2\left(\frac{a}{18}\right)^2 + \frac{aa^3}{12} + \frac{a^2}{2}\left(\frac{a}{4}\right)^2$$

$$I_{\bar{y}\bar{y}} = I_{\overline{y_1y_1}} + I_{\overline{y_2y_2}} + I_{y_1y_1} + A_1\Delta y_1^2 + I_{y_2y_2} + A_2\Delta y_2^2 = \frac{aa^3}{12} + a^2\left(\frac{5}{18}a\right) + \frac{aa^3}{12} + \frac{a^2}{2}\left(\frac{5}{9}a\right)$$

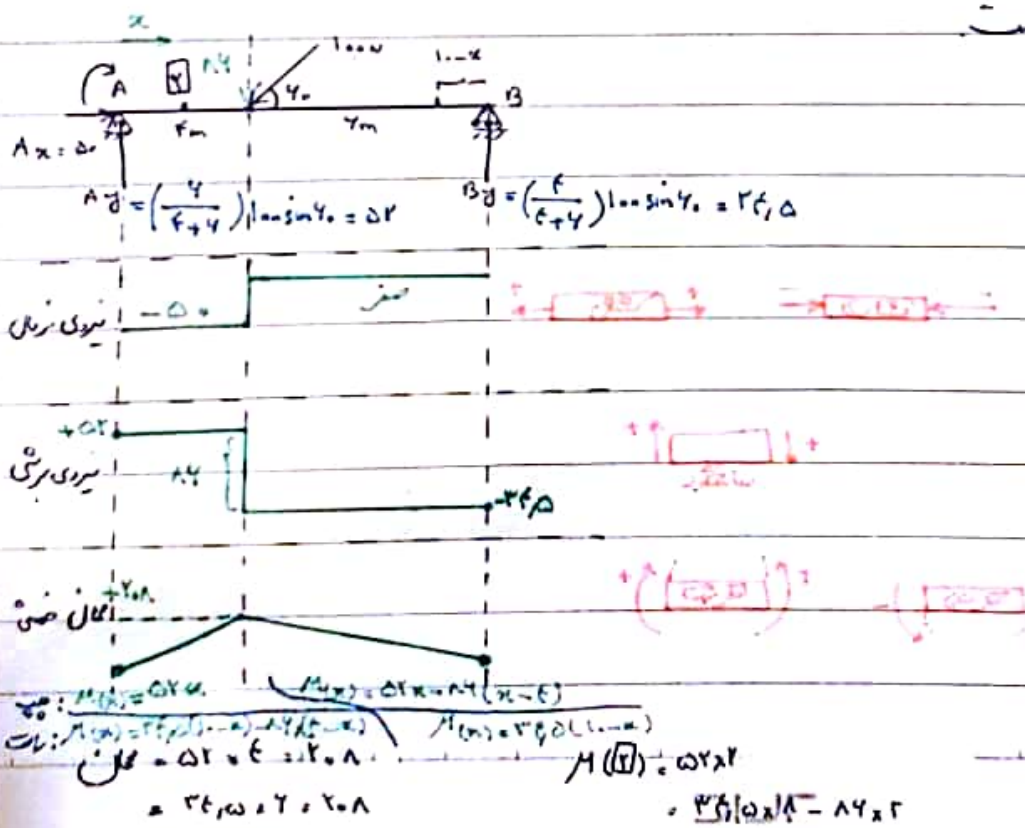
$$I_{\bar{x}\bar{x}} = 0.112a^4 \qquad I_{\bar{y}\bar{y}} = 0.364a^4$$

نیروی برشی: /

نیرویی که عمود بر سطح مقطع است.

نکته: هر جا مفصل بود، مان صفر است!

[سؤال: برای تیر مفروض، دیاگرام نیروی برشی، نیروی برشی و مان خمشی را بدست آورید؟]



$M(x) = 520x$

$M(x) = 520x - 866(x-4)$

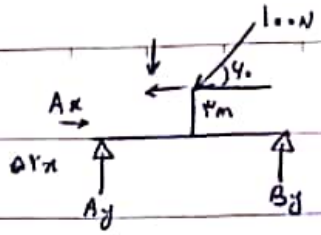
$M(x) = 346.5(1-x)$

$M(4) = 520 \times 4 = 2080$

$M(2) = 520 \times 2 = 1040$

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حل مسئله تیر با ارتفاع:



$\Sigma F_x = 0 \rightarrow A_x = 50$

$\Sigma F_y = 0 \rightarrow A_y + B_y = 85$

$\Sigma M_A = 0 \rightarrow F(10 \sin 60) - 2(10 \cos 60) - 10 B_y = 0 \rightarrow B_y = 19 \rightarrow A_y = 66$

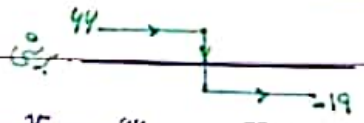
نیروی محوری



$N(x) = -50 \quad 0 < x < 4$

$N(x) = 0 \quad 4 < x < 10$

برشی



$V(x) = 66 \quad 0 < x < 4$

$V(x) = -19 \quad 4 < x < 10$

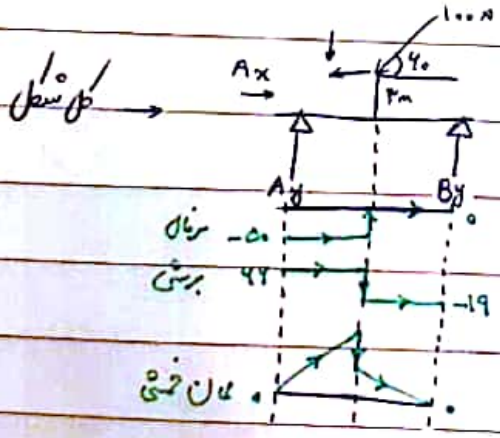
لنگر خمشی

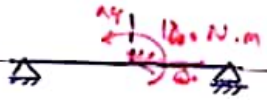


$M(x) = 66x \quad 0 < x < 4$

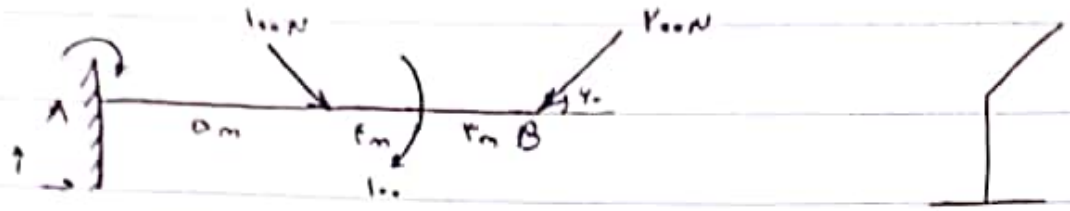
$M(x) = 19(10 - x) \quad 4 < x < 10$

کل سطل





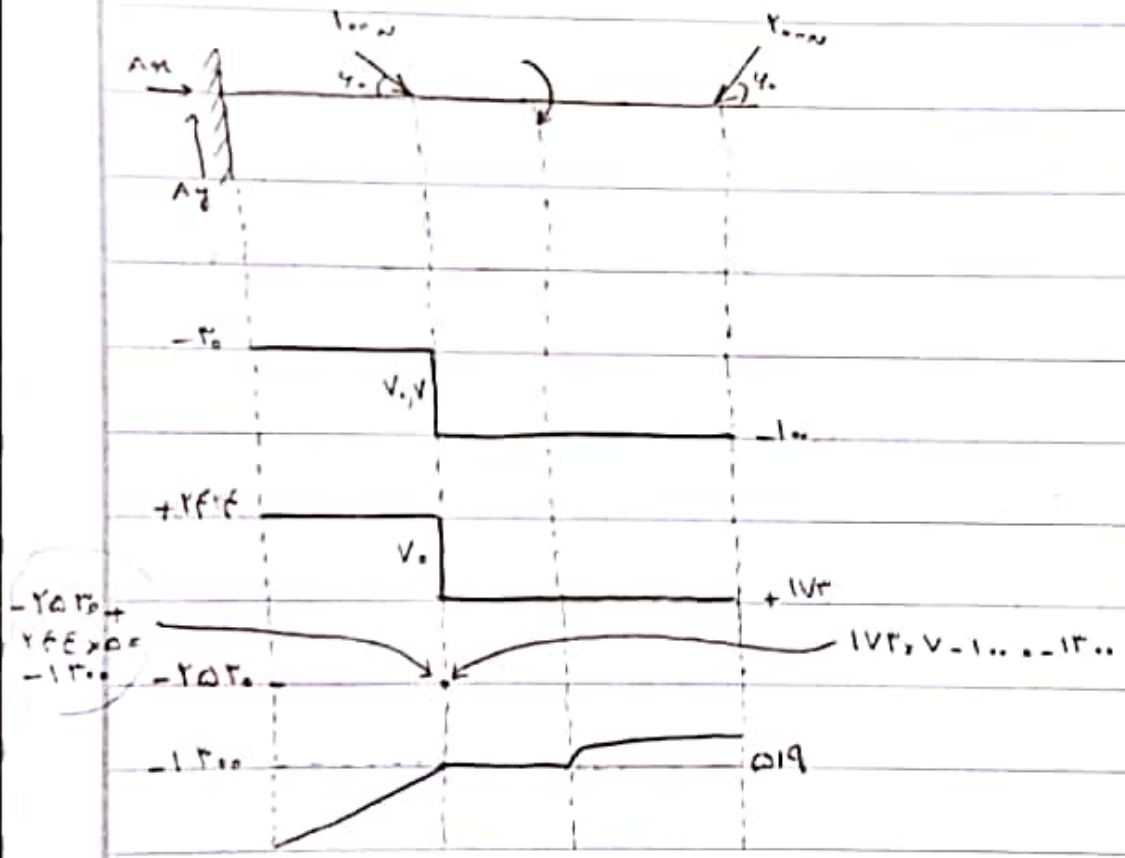
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$\Sigma F_{x=0} \rightarrow 100\cos 53 - 200\cos 45 + Ax = 0 \rightarrow Ax = 73$

$\Sigma F_{y=0} \rightarrow 100\sin 53 - 200\sin 45 + Ay = 0 \rightarrow Ay = 221$

$\Sigma M_A = 0 \rightarrow 100\sin 53\,(0) + 100 + 200\sin 45\,(12) + M_A = 0 \rightarrow M_A = -2053$



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Date



۱. مثال: انیروها؟

۲. حل) تنش جسم ۲ مقدار است از سطح مقطع آن $V cm^2$ باشد؟

$$\sigma = \frac{F}{A} = \frac{V000}{V} = 1000 \ \frac{kg.f}{cm^2}$$

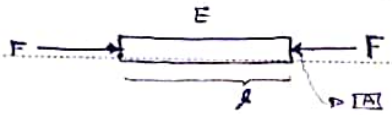
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(جلسه دهم)

کرنش، مدول الاستیسیته، تنش

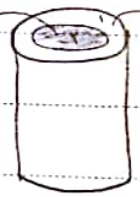
$$\sigma = E \cdot \varepsilon \qquad \sigma = \frac{F}{A} \quad \varepsilon = \frac{\Delta l}{l} \qquad \frac{F}{A} = E \cdot \frac{\Delta l}{l}$$

$$F \propto l$$



مثال) داخل یک لوله فولادی به سطح مقطع $6\,cm^2$ و مدول $2\times10^6\ \frac{kg}{cm^2}$ از بتن پر شده است. سطح مقطع بتن $24\,cm^2$ و مدول آن $2\times10^5\ \frac{kg}{cm^2}$ است.

پس از وارد آوردن نیروی P سهم این مجموعه نسبت به نیروی داخل فولاد و بتن چقدر است.



$$\boxed{I} \rightarrow \Delta l_1 = \Delta l_2 \xrightarrow{\text{طبق}} \frac{F}{A} = E \cdot \frac{\Delta l}{l}$$

$$\rightarrow \frac{F_1 l_1}{E_1 A_1} = \frac{F_2 l_2}{E_2 A_2} \xrightarrow{\text{چون}} l_1 = l_2$$

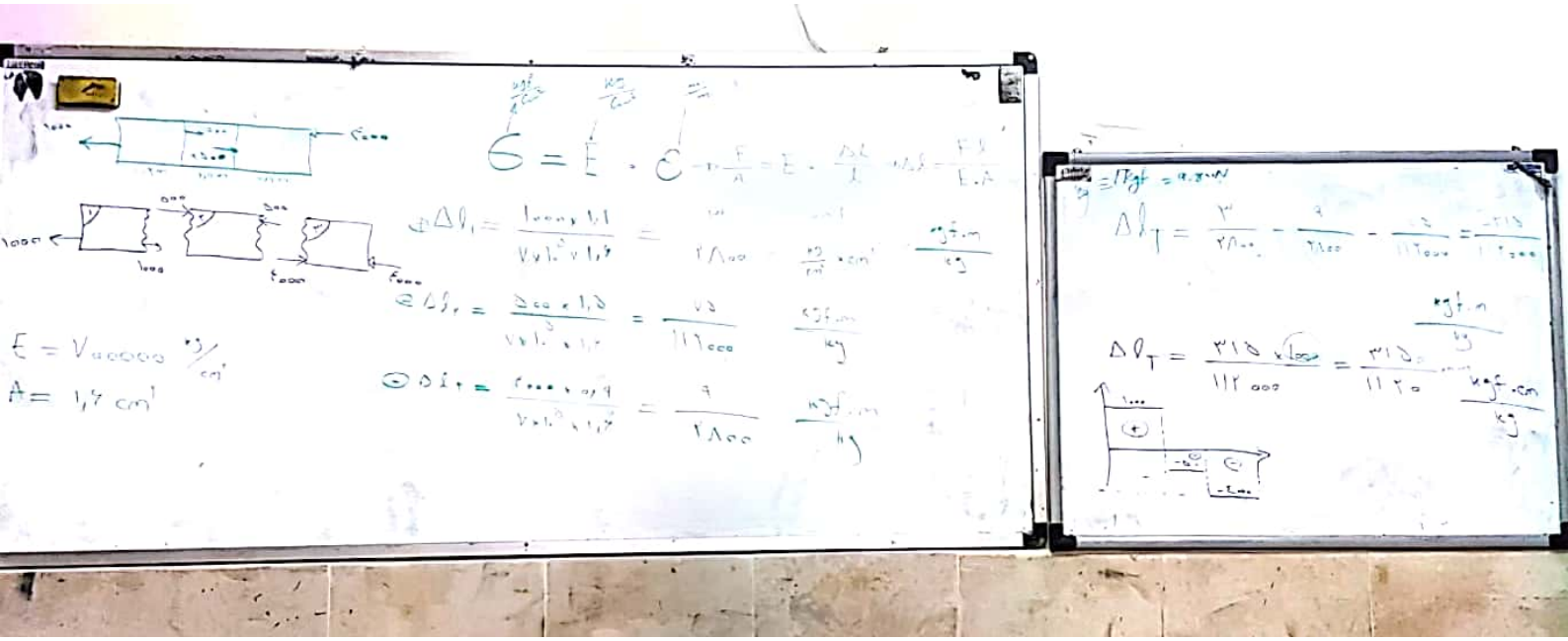
$$\Rightarrow \frac{F_1}{E_1 A_1} = \frac{F_2}{E_2 A_2} \rightarrow \frac{F_1}{F_2} = \frac{2\times10^6 \times 6}{2\times10^5 \times 24} = \frac{10}{4} = \frac{5}{2} = 2.5$$

به ازای اینکه به فولاد 2.5 برابر بتن نیرو وارد شود، یک ضریبی از این مجموعه را میتوان وارد کرد تا موضع به تسلیم برسد.

* در این سوال شرط نبود پیوستگی است. Δl_1 با Δl_2 برابر است پس $\boxed{I}$

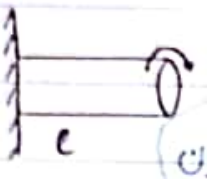
PARCO

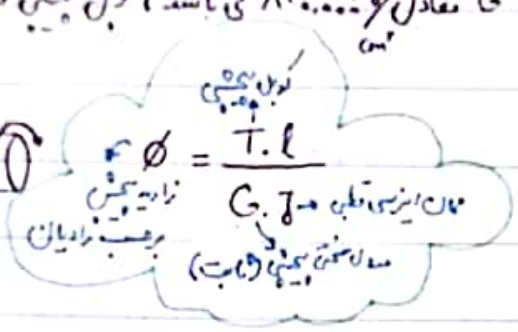
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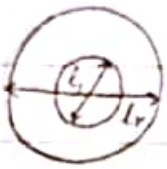
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مثال) میله‌ای که قطر خارجی آن ۱۰۰ و قطر داخلی آن ۵۰ می‌باشد و جنس آن میله، G معادل $8400000\ kg/cm^2$ می‌باشد، گشتاور پیچشی معادل ۱٫۲ kN.m





اعمال می‌گردد. اگر طول لوله ۱m باشد، سر لوله چند درجه خواهد چرخید؟



$l_1 = 50\ cm = \frac{1}{2}\ m$

$l_2 = 100\ cm = 1\ m$

$G = 8400000\ kg/cm^2 = 84 \times 10^8\ kg/m^2$

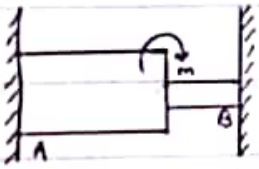
$T = 1.2\ kNm$

$J = (l_2^4 - l_1^4) \times \frac{\pi}{32}\ (m^4) = (1 - \frac{1}{16}) \times \frac{\pi}{32} = \frac{15}{16} \times \frac{\pi}{32}\ m^4$ ← واحد

$\phi = \frac{1.2 \times 10^3 \times 1}{84 \times 10^8 \times \frac{15}{16} \times \frac{\pi}{32}} = \frac{1200 \times 16 \times 32}{10^8 \times 84 \times 15 \times 3.14} = \frac{614400}{10^8 \times 3954} = 155 \times 10^{-8} \times \frac{18}{\pi}$

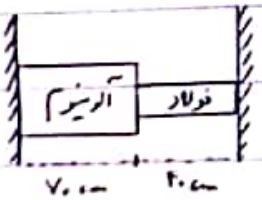
$x_{\text{واحد}} = \frac{T.l}{G.J} \rightarrow \frac{Nm.m}{\frac{kg}{m^2}.m^4} = \frac{1 \times kg\,m^4}{kg.m^4}$

مثال) در شکل زیر اگر طول و جنس میله A و B یکسان ولی قطر میله‌ی A ۲ برابر قطر میله‌ی B باشد، نسبت گشتاور پیچشی میله A و B چقدر است؟



$\frac{\phi_A}{\phi_B} = \frac{\frac{T_A.l_A}{G_A.J_A}}{\frac{T_B.l_B}{G_B.J_A}} \rightarrow$

مثال) میله‌ای از دو قسمت آلومینیوم و فولاد ساخته شده و مطابق شکل زیر بین دو دیوار که به صورت صلب در نظر گرفته می‌شوند، محکم شده است. در فصل مشترک این دو فلز، نیروی محدودی $P = 2100\ kgf$ اعمال می‌شود. مطلوب است نسبت نیروی فولاد به آلومینیوم؟



$E_{Al} = 0.7 \times 10^7\ \frac{kgf}{cm^2}$

$A_{Al} = 24\ cm^2$

$E_{st} = 2 \times 10^7\ \frac{kgf}{cm^2}$

$A_{st} = 12\ cm^2$

$\frac{F}{A} = E.\frac{\Delta l}{l} \rightarrow \frac{F_{Al}\,L_{Al}}{A_{Al}\,E_{Al}} = \frac{F_{st}\,L_{st}}{A_{st}.E_{st}}$

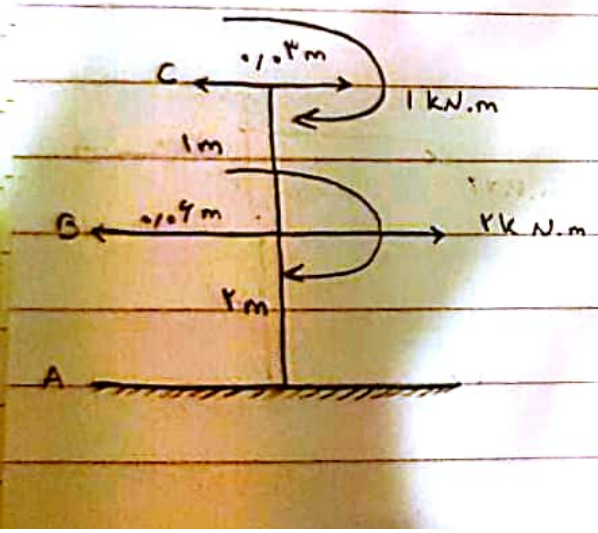
$\frac{F_{Al} \times 70}{24 \times 0.7 \times 10^7} = \frac{F_{st} \times 40}{12 \times 2 \times 10^7} \rightarrow F_{Al} \times 4.17 = F_{st} \times 1.67$

$\frac{F_{st}}{F_{Al}} = \frac{4.17}{1.67} = 2.50$

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ماکزیمم زاویه پیچش در نقطه A را بدست آورید؟ $J_{AB} = 1,272 \times 10^{-6}\ m^4$ و $J_{BC} = 0.0796 \times 10^{-6}\ m^4$

و $G = 80\ GPa$ می باشد.



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